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Mechanism Design for "Free" but "No Free Disposal" Services: The Economics of Personalization Under Privacy Concerns

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Online personalization services belong to a class of economic goods with a "no free disposal" (NFD) property where consumers do not always prefer more services to less because of the privacy concerns. These concerns arise from the revelation of information necessary for the provision of personalization services. We examine vendor strategies in a market where consumers have heterogeneous concerns about privacy. In successive generalizations, we allow the vendor to offer a fixed level of personalization, variable levels of personalization, and monetary transfers (coupons) to the consumers that depend on the level of personalization chosen. We show that a vendor offering a fixed level of personalization does not offer a coupon unless his marginal value of information (MVI) is sufficiently high, and even when personalization is costless, the vendor does not cover the market. Under a fixed services offering, the vendor serves the same market with or without couponing. Next, we demonstrate that in the absence of couponing, the vendor's optimal variable personalization services contract maximizes surplus for all heterogeneous consumers, which is in contrast to standard results from monopolistic screening. When the vendor can offer coupons that vary according to personalization levels, the optimal contract is not fully revealing unless his MVI is high and he will not offer coupons when this MVI is low. However, a vendor with a moderate MVI (between certain thresholds) offers a bunched contract, wherein consumers with low privacy concerns receive a variable services-coupon contract, those with moderate privacy concerns receive a fixed services-coupon contract, and those with high privacy concerns do not participate in the market. The coupon value is decreasing in privacy sensitivity of consumers.

Key words: incentives; mechanism design; personalization; privacy; no free disposal
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1. Introduction

Recent advances in Internet technologies have allowed online vendors to provide personalization to users of both computers and mobile devices. Whereas personalization in computers is typically accomplished through browser-embedded toolbars, deskbars, or sidebars, personalization in mobile devices including Smartphones and personal digital assistants is normally accomplished through the homescreen and other embedded widget technologies. These technologies rely on user-supplied usage information to provide various personalized services. For example, a toolbar is a type of browser helper object (BHO), which, once downloaded and embedded, has the ability to monitor and report usage information (including websites visited, information completed in online forms, etc.) as well as to tailor future Web browsing. Generally, these personalization technologies and services accompanying them are available free of charge because the vendor's profit motive is based on exploiting consumers' preference information acquired during the usage of these services. On the flip side, consumers make a trade-off between the conveniences these "free" personalization services offer and the breach of privacy that results in sharing personal and preference information required to use these services. Although online vendors have been focused on getting consumers to share information through personalization offerings, the question, "does personalization jeopardize our privacy?" has been of great interest to the legal community and public at large (Volokh 2000, p. 84).

1.1. Delivery of Online Personalization Services

Personalization services belong to a less-studied and special class of economic goods and services that exhibit the "no free disposal" (NFD) property, i.e.,

goods that have an intrinsic disutility from consumption where more is not necessarily better. Classic examples include identifying the right package size (larger sizes are not always optimal) of travelers' toiletries (Nahata et al. 2006), ideal level of sweetness of a drink (over sweetness may not be preferred), and the optimal units of power production (production creates pollution) for a township (Rothwell and Rust 1997). Free disposal is often a central assumption in many economic models because it is technically convenient (Repullo 1987), but more recently computational approaches in auctions research have examined the impact of selling NFD goods wherein an auctioneer incurs some cost to keep unsold goods (Sandholm and Suri 2003).

When consumers are heterogeneous in their privacy costs with different satiation levels for personalization services, the vendor's services-related choices are limited because of the lack of price instruments and hidden privacy costs of the consumers. Thus, in responding to the Rust et al. (2002) observation that with increased technological sophistication a market for privacy is emerging, any mechanism design for such markets needs to explore nonprice instruments (Murthi and Sarkar 2003). Indeed, researchers have noted the importance of providing subsidies or side payments to get consumers to share information (Resnick and Varian 1997), and thus raising the potential for couponing as an instrument in this market. There is clear evidence of online firms experimenting with these incentives alongside personalization. For example, in the pretoolbar era, Amazon introduced coupons in the form of discount coupons based upon consumers' clicking through its Gold Box services and sharing certain information. More recently, Microsoft announced plans to launch a program called "Live Search cashback" that reportedly will share a percentage of its revenues if the user shares purchase information through its Microsoft Live services (Bishop 2008).

Our paper models the optimal services-coupons strategies of a personalization vendor whose objective is to maximize information acquisition in a market where consumers are heterogeneous in their privacy costs, known only to the consumer. The vendor engages the market through either a fixed level or menu of services, along with the option to offer fixed or variable coupons. Fixed services provide a fixed number of personalization services in return for a fixed amount of information—a consumer typically has to install the whole toolbar or not use it at all. A9.com's (owned by Amazon.com) toolbar is a classic example of such a fixed offering. The variable services strategy allows consumers to choose a subset of personalization services (and thus sharing only the corresponding information)—Google's own toolbar is an example.

1.2. Review of Relevant Literature

In prior work on mechanism design, there is no specific stream of research that has studied NFD goods, with some exceptions in computational approaches to auction design (Sandholm et al. 2002). However, in the area of digital goods/services, recent research has studied pricing of databases (Jain and Kannan 2002), application service providers (Cheng and Koehler 2003), and information goods (Geng et al. 2005, Shapiro and Varian 1998). Usage-based pricing is a common theme among researchers in information systems where price is a strategic variable (Mendelson and Whang 1990), and is generally known to be superior to fixed offerings (Maskin and Riley 1984). In this vein, Sundararajan (2004) observes that a unique feature of information goods, namely zero marginal cost, can lead to alternative optimal pricing strategies. Because the optimal personalization strategy is intrinsically related to hidden privacy costs of a consumer, our research question of finding an optimal menu of services-coupon approaches that of a monopolist's screening problem with hidden ability (Baron and Myerson 1982). To that extent, effort in the standard screening problem is analogous to personal information in our model, and although different employees bear different costs of effort according to their ability in standard screening problems, different users bear different privacy costs according to their privacy sensitivity in our model. And coupons are like wages, contingent on observed use of services like observed output. On the other hand, our monopolist vendor's objective function is different—the vendor here is "buying" personal information and trying to screen consumers on account of not knowing their cost of giving up personal information. Furthermore, whereas the consumers' utility in a standard screening model is monotonic in effort, i.e., more effort is always more or at least as costly, in our model because the consumer gets an intrinsic personalization benefit from sharing information, his utility first increases and then falls in services consumed (information shared) due to privacy costs. In summary, there are two critical differences between our model and standard monopoly screening problems examined in economics literature, namely the NFD property of personalization services and the inability of a vendor to charge prices for these services.

Our analysis results in some counterintuitive findings. First, in many cases, the vendor does not serve the entire market even if it is costless for him to do so. Second, a vendor pursuing a fixed services strategy serves the same portion of the market with or without a coupon, suggesting that coupons are not useful in converting privacy seekers. Third, in a variable services-only strategy, the vendor offers

consumer-surplus-maximizing services thus underlining the need for additional instruments for NFD services so that the vendor may capture some of the consumer surplus. Finally, in the case of variable services-coupons strategy market coverage depends on the vendor's marginal value of information (MVI). Whereas a vendor with high MVI serves the entire market with a services-coupons menu, a vendor with moderate MVI offers a bunched contract and does not serve privacy seekers because of the NFD property and the nonprice nature of the market. A vendor with low MVI again serves the entire market but offers no coupons.

This paper is organized as follows: In §2, we set up the basic model and objective functions. In §3, we examine different vendor strategies and develop a bunched solution as the optimal contracting mechanism. In §4, we discuss our results and identify suitable theoretical and managerial implications.

2. Model

Our model consists of a monopolist vendor who provides free personalization services and consumers who use these services. Consumers evaluate their benefits of disclosure against the risks (Culnan and Armstrong 1999) resulting in a NFD utility in personalization services. Generically, this implies that the consumers' utility is decreasing in services consumed after a satiation level, leading to an inverted U-shaped function (i.e., nonmonotonic) in services.

We consider a market where consumers are homogeneous in their marginal value for personalization, $p > 0$, and heterogeneous in their concerns for privacy such that their privacy cost parameter $r \in [\underline{R}, \bar{R}]$ is distributed with density function $f(r)$ and cumulative density $F(r)$ that is continuously differentiable. Consumers with a higher value of r are more privacy sensitive. Thus, for the most privacy-sensitive consumer (privacy seeker), $r = \bar{R}$, and for the least privacy-sensitive consumer (convenience seeker), $r = \underline{R}$. Furthermore, $f(r)$ is assumed to be single peaked (unimodal) and is everywhere positive on its support such that its hazard function $h(r) = f(r)/F(r)$, satisfies the monotone hazard rate property, i.e., $h'(r) \leq 0$. Most common distributions satisfy these assumptions.

Prior research (Chellappa and Shivendu 2006, 2007) has modeled consumer behavior in this context through a nonmonotonic utility of the form

$$u(r, s) = ps - r[\psi s]^2,$$

where s: $s \in \mathbb{R}^+$ is the level of personalization services and ψs is the preference information the consumer shares as a result of using s level of personalization services. The number of personalized services that can be created from a unit of information is a function of the prevalent personalization and data-mining technologies (Raghu et al. 2001), and $\psi > 0$ is a personalization-technology parameter that reflects difficulty in offering personalization with a given set of information, or how readily information can be converted into personalization—a lower ψ reflects a more advanced technology and thus a greater level of personalization for the same level of information revealed (Chellappa and Shivendu 2006).

In summary, we assume the following.

1. Consumers' utility is nonmonotonic concave in the services consumed:

$$u(r, s) = ps - r[\psi s]^2.$$

2. Critical services levels are defined as

$$s_c^*(r) = \arg\max_s \{u(r, s)\} = \frac{p}{2\psi^2 r} \quad \text{and}$$

$$s_c^0(r) = \arg_s \{u(r, s) = 0\} = \frac{p}{\psi^2 r}.$$

3. The vendor cannot charge prices because personalization services are free.

4. Versioning the personalization services is costless for the vendor.

5. The marginal cost of serving an additional consumer as well as the cost of administering services is zero because personalization technology automates information acquisition.

2.1. Vendor

The vendor of the personalization service is a monopolist who chooses the nature of the personalization services (henceforth the "services") offered as well as whether to offer consumers positive monetary transfers (often called coupons) in conjunction with the services. Although there are no direct monetary payments from the consumers to the vendor, the vendor receives personal information from each consumer who adopts the services, and the vendor is assumed to be able to derive value from this information.

More formally, suppose the vendor's optimal offer results in consumers of type r adopting the service at a personalization level $s(r)$ and receiving a coupon $t(r)$ in exchange for providing personal information $\psi s(r)$. Each consumer of type r will make the same choice. Furthermore, the revelation principle ensures that we can restrict our attention to direct mechanisms, reflected in the formulation that follows. If it turns out that a consumer of type r does not adopt the service, then $s(r) = t(r) = 0$. The surplus of a consumer of type r is thus

$$ps(r) - r\psi^2[s(r)]^2 + t(r),$$

and we model the payoff to the vendor as

$$\alpha\left[\int_{\underline{R}}^{\bar{R}} \alpha s(r) f(r)\, dr\right] - \int_{\underline{R}}^{\bar{R}} t(r) f(r)\, dr.$$

The first term in the vendor payoff function represents the value the vendor derives from the information obtained collectively from the consumers. One might think of the parameter α as representing the vendor's MVI. The second term represents the total monetary transfers from the vendor to the consumers.

We consider increasingly general instances of this model in succession, contrasting a situation in which the vendor cannot customize his services and must offer each consumer the same level of personalization ("fixed services") with the case in which the vendor may offer a menu of personalization levels ("variable services"), and situations in which the vendor cannot offer any monetary transfers to the consumers ("no coupons") with the ability to offer a transfer of fixed value to all adopting consumers ("fixed coupons") and the ability to make the coupon value contingent on the level of personalization ("variable coupons"). We use the subscripts *fs* and *vs* to denote fixed services and variable services, respectively, and we use the subscripts *nc*, *fc*, and *vc* to represent the cases of no coupons, fixed coupons, and variable coupons.

Consider the case where the vendor offers a fixed service and no coupons (*fs–nc*). It follows that $s(r) = s$ for each r, and $t(r) = 0$ for each r. Let $r_{fs\text{–}nc}$ be the most privacy-sensitive consumer type who adopts this fixed service (we will show later that this adoption has a monotonicity property). The vendor's optimization problem is therefore to choose the level of personalization s that solves

$$\max_{s} \alpha\left[\int_{\underline{R}}^{r_{fs\text{–}nc}} \alpha s f(r)\, dr\right],$$

subject to the participation constraint

$$ps - r\psi^2 s^2 \geq 0$$

for each $r \in [\underline{R}, r_{fs\text{–}nc}]$. Similarly, if the vendor offers a fixed service with a fixed coupon (*fs–fc*), then $s(r) = s$ and $t(r) = t$ for each r. If $r_{fs\text{–}fc}$ is the most privacy-sensitive consumer type who adopts, the vendor's optimization problem involves choosing the pair (s, t) that solves

$$\max_{s,\,t} \alpha\left[\int_{\underline{R}}^{r_{fs\text{–}fc}} \alpha s f(r)\, dr\right] - t,$$

subject to the participation constraint

$$ps - r\psi^2 s^2 + t \geq 0$$

for each $r \in [\underline{R}, r_{fs\text{–}fc}]$.

Next, if the vendor offers variable services but with no couponing (*vs–nc*), $t(r) = 0$ for each r. Let $r_{vs\text{–}nc}$ be the most privacy-sensitive consumer type who adopts the personalization service. The vendor chooses the revenue-maximizing menu of services $s(r)$ that solves

$$\max_{s(r)} \alpha\left[\int_{\underline{R}}^{r_{vs\text{–}nc}} \alpha s(r) f(r)\, dr\right],$$

subject to the participation constraint

$$ps(r) - r\psi^2[s(r)]^2 \geq 0$$

and the incentive-compatibility constraint

$$s(r) = \arg\max_{s}[ps - r\psi^2 s^2]$$

for each $r \in [\underline{R}, r_{vs\text{–}nc}]$.

Finally, suppose the vendor can offer coupons that depend on the level of personalization (*vs–vc*). Define

$$U(r) = ps(r) - r\psi^2[s(r)]^2 + t(r)$$

as the consumer surplus of a consumer of type r in this case. It follows that the vendor chooses the menu $s(r), t(r)$ that solves the following problem:

$$\max_{s(r),\,t(r)} \left[\int_{\underline{R}}^{r_{vs\text{–}vc}} [\alpha s(r) - t(r)] f(r)\, dr\right],$$

subject to the participation constraint

$$U(r) \geq 0$$

and the incentive-compatibility constraint

$$U(r) \geq \max_{x \in [\underline{R},\, r_{vs\text{–}vc}]} [u(r, s(x)) - t(x)]$$

for each $r \in [\underline{R}, r_{vs\text{–}vc}]$, where $r_{vs\text{–}vc}$ is the most privacy-sensitive consumer type who adopts the services.

3. Vendor Strategies Under Information Asymmetry

3.1. Fixed Services Contracts

Lemma 1. *In any fixed services contract* (*with or without coupons*) *the vendor serves only a portion of the market. When couponing is employed, the value of the coupon is increasing in the level of service offered.*

When the vendor offers a fixed services contract, all consumers who derive positive utility for that level of services participate. Let $r_{fs\text{–}nc}$ be the consumer who is indifferent between taking the contract and not participating. Because $\partial u(r, s)/\partial r < 0$, all consumers whose privacy sensitivity is less than $r_{fs\text{–}nc} = p/(\psi^2 s)$ individually contract, and the market given by $r \in [\underline{R}, r_{fs\text{–}nc}]$

is served. The optimal fixed services offered without coupons is

$$s^*_{fs-nc} = \frac{p}{\psi^2} h(r^*_{fs-nc}),$$

and fixed services offered with coupons is

$$s^*_{fs-fc} = \frac{p + \alpha\psi}{2\psi^2} h(r^*_{fs-fc}).$$

From the optimal service level, we observe $r^*_{fs-nc} \geq$ mode of $f(r)$, implying that for any symmetric unimodal distribution, a fixed services contract will serve more than half of the market, leaving out the consumers with higher privacy costs. The portion of the market served is smaller if the market consists predominantly of convenience seekers (right-skewed distribution) than one consisting largely of privacy seekers (left-skewed distribution), though the profits to the vendor are higher in the former. We also can see that as the personalization technology improves (ψ decreases), the services levels, market served, and the vendor profits are all increasing.

Commonly in price contracts for free disposal goods/services, even if all consumers derive positive value from usage of services, prices may eliminate some consumers in the market because of differences in their reservation valuations. Although there are no prices in our model, the NFD property is the source of some consumers being eliminated from the market, i.e., there may be consumers for whom the privacy cost may be higher than the personalization benefit at the offered services level. In addition, even when the vendor has no costs of serving consumers, it is not optimal for him to offer the entire set of feasible services. The intuition behind this is that the vendor is capped by the maximum services ($s^0_c(\underline{R})$) that can be consumed (by the least privacy-sensitive consumer) in this market because of the NFD property of the services. Thus, in determining his optimal offering, a vendor considers the trade-off between increasing the market size by reducing the fixed services level and increasing the information extracted from each consumer by increasing the fixed services level.

From the analysis of Lemma 1, it is unclear when the vendor should choose the option of offering a fixed coupon along with fixed services. When the vendor offers fixed coupons, r_{fs-fc} is the consumer type that is indifferent between participating and not and is given by

$$r_{fs-fc} = \frac{ps_{fs-fc} + t_{fs-fc}}{\psi^2 s^2_{fs-fc}}.$$

Consumers with privacy sensitivity less than r_{fs-fc} participate.

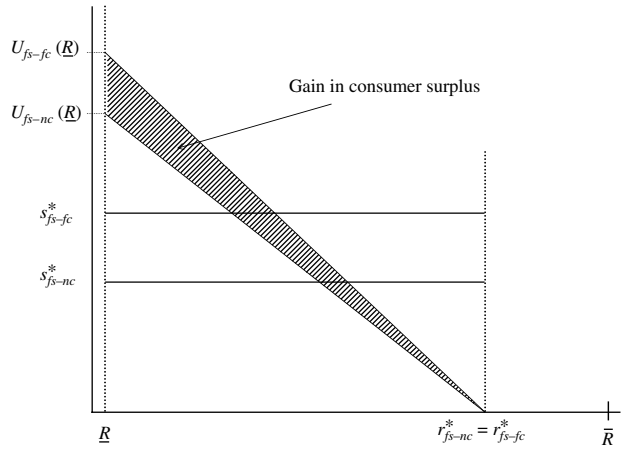
Proposition 1. *In the fixed services offering, fixed couponing is an optimal strategy for a vendor with sufficiently high MVI* ($\alpha\psi \geq p$). *Consumer surplus and social welfare are also higher under fixed couponing though the same set of consumers are served with or without a coupon* ($r^*_{fs-nc} = r^*_{fs-fc}$). *It is never optimal for the vendor to offer a fixed-services variable-coupons contract.*

This proposition provides an interesting characterization of a vendor who cannot vary personalization (for example, who provides a personalization toolbar of fixed functionality) and might consider offering a coupon to incentivize the adoption of the toolbar. Intuitively, it might appear that providing an additional fixed coupon will increase market size; however, counter to intuition, the vendor serves the same market as in the absence of any coupon. The economic rationale behind this observation is that a fixed coupon increases the break-even services level for all consumers, and the vendor can extract this surplus by either offering the same services level as in Lemma 1 and getting a larger portion of the market to participate, or by increasing the services level for the same market and compensating each consumer such that the last consumer (of type r^*_{fs-nc}) still gets zero utility. Choosing between the two options depends on the vendor's marginal gain from additional services versus the marginal gain from increased market size. Because the fixed coupon only changes the intercept of consumer utility without affecting this trade-off, the vendor opts for the higher services level and compensates the same consumer base with a fixed coupon.

The results also provide an intuitive separation condition between when a coupon is offered and when it is not—this depends on whether the MVI, α, is larger or smaller than the ratio of the marginal "direct value of personalization" and the "difficulty of personalization," p/ψ. Intuitively, a vendor with a higher MVI can afford to offer coupons in exchange for information, whereas a vendor with a lower α simply adjusts the level of personalization instead. This trade-off skews toward adjusting personalization when its direct value of personalization is higher and skews toward couponing when the difficulty of personalization is higher.

The producer surplus is higher with couponing because the marginal value from the services is higher for the vendor compared with the value to the consumers ($\alpha\psi > p$). Also, he can give back a portion of this surplus to each consumer as a coupon and still be left with positive gains. Under information asymmetry, the vendor is able to extract full surplus only from the last participating type, and all others are left with a residual gain that cannot be extracted (as shown in Figure 1). Therefore, couponing leads to increased consumer welfare, producer surplus, and hence social welfare, though the market size remains the same. Note that we consider a fixed coupon offering only with a fixed services contract. If some level of

Figure 1 Gains from Couponing in Fixed Contracts



fixed services s_f is offered with variable coupons $t(r)$, a consumer will always misrepresent his type such that his utility $ps_f - r\psi^2 s_f^2 + t(r)$ is maximized. In fact, all consumers will declare the same $\tilde{r}$ at which $t(r)$ is maximized, leaving the vendor worse off.

3.2. Variable Services Contracts

Lemma 2. (*vs–nc*): *In a variable services-only contract the vendor will serve the entire market, and the contract is given by* $s^*_{vs-nc}(r) = p/2\psi^2 r$.

Proposition 2. *The variable services-only contract is consumer surplus maximizing. It is never optimal for the vendor to offer a variable-services fixed-coupon contract.*

A vendor's offering of a menu of services but no coupons is illustrative of the price-free but NFD nature of personalization services. Though in the full information case, a vendor can offer $s_c^0(r)$ to each consumer of type r and extract full consumer surplus, under information asymmetry the vendor offers a menu in which consumers select the services levels. For any $s(r)$ offered by the vendor, the consumers declare a $\tilde{r}$, such that $s(\tilde{r}) = p/2\psi^2 r$. Hence, the vendor has no option but to offer $s(r) = p/2\psi^2 r$, because this is the only way the menu can be truth revealing and incentive compatible.

It is perhaps surprising that a monopolist vendor finds a consumer-surplus-maximizing contract to be optimal. This result stems from two aspects of market: the NFD property of the personalization services and the inability of the vendor to charge prices. For any nonmonotonic utility function that satisfies the first assumption in §2.1, a consumer of type r can always misrepresent himself as some $\tilde{r}$ such that $u(r, s(\tilde{r})) = \max_s u(r, s)$ for any services level. The vendor is unable to curb this behavior because he has no strategic instrument at his disposal and has to manage the information asymmetry in the market through services alone. Hence, his best option is to offer the consumer-surplus-maximizing services and let consumers pick their optimal levels.

Note that, if the vendor were to offer a variable-services fixed-coupon contract, the consumer will falsely declare $\tilde{r} = 2r$ and will get the fixed coupon as well. Thus, it is never optimal for the vendor to offer a fixed coupon with a variable services offering, not because of the NFD property of the services, but rather because of the inability of the vendor to charge prices in this market. In price-quantity schedules, vendors set prices such that the consumers are incentivized to buy more, i.e., higher quantities imply higher discounts or lower prices (the two move in opposite directions). Similarly in price-quality schedules, higher quality implies higher prices, i.e., the two move in the same direction; the fundamental requirement being that both instruments have to be related such that if one changes the other also must change. If only one instrument is available to the vendor, then agents will always falsify their privacy costs so as to maximize their rents, and the vendor will be unable to design a truthful direct-revelation mechanism.

Now we consider the case in which the vendor can offer a menu of services-coupons to incentivize consumers to use more services. Let the vendor offer a variable-services variable-coupons (*vs–vc*) contract $\{s_{vs-vc}(r), t_{vs-vc}(r)\}$ that is selected by a consumer of type r. Before we solve the vendor's optimization problem, we first must find conditions for which the contracts will be truth revealing. Suppose a consumer of type r can declare some $\tilde{r}$; then for the mechanism to be truthful, the consumer should not be worse off by declaring his type as r. The condition for truthfulness is givens as

$$\begin{aligned} U(r) \geq U(\tilde{r}) \quad \Rightarrow \quad & t(r) + ps(r) - \psi^2 r[s(r)]^2 \\ & \geq t(\tilde{r}) + p(\tilde{r}) - \psi^2 r[s(\tilde{r})]^2 \end{aligned} \tag{1}$$

for any $(r, \tilde{r})$ in $[\underline{R}, \bar{R}] \times [\underline{R}, \bar{R}]$. Equation (1) implies that for all pairs $(r, \hat{r})$ in $[\underline{R}, \bar{R}] \times [\underline{R}, \bar{R}]$, we need

$$\begin{aligned} & t(r) + ps(r) - \psi^2 r[s(r)]^2 \geq t(\hat{r}) + ps(\hat{r}) - \psi^2 r[s(\hat{r})]^2, \\ & t(\hat{r}) + ps(\hat{r}) - \psi^2 \hat{r}[s(\hat{r})]^2 \geq t(r) + ps(r) - \psi^2 \hat{r}[s(r)]^2. \end{aligned} \tag{2}$$

Adding the above two equations in (2), we get

$$\begin{aligned} \hat{r}(s(r) - s(\hat{r})) \ \geq \ & r(s(r) - s(\hat{r})) \\ \Rightarrow \ & (\hat{r} - r)(s(r) - s(\hat{r})) \geq 0. \end{aligned} \tag{3}$$

Thus, the incentive-compatibility constraint requires (as in (3)) that the schedule of services $s(r)$ must be nonincreasing, and should also meet the almost everywhere differentiability conditions for both the services and coupons schedules. From (1),

we know that in order for response $\tilde{r}$ to be optimal for a consumer type r,

$$t'(\tilde{r}) + ps'(\tilde{r}) - 2\psi^2 rs(\tilde{r})s'(\tilde{r}) = 0,$$

using prime and double prime notation for the first- and second-derivative, respectively. For truth to be an optimal response for type r, it must be the case that

$$t'(r) + ps'(r) - 2\psi^2 rs(r)s'(r) = 0. \quad (4)$$

Equation (4) must hold for all r in $[\underline{R}, \bar{R}]$ because r is unknown to the vendor. We also need to ensure that the menu satisfies both local and global incentive-compatibility constraints and that the latter is ensured by the constant-sign or Spence-Mirrlees condition. To satisfy the former, we need to show that the services menu is nonincreasing in privacy costs ($s'(r) \le 0$), i.e., satisfies the local second-order condition:

$$t''(r) + ps''(r) - 2r\psi^2([s'(r)]^2 + s(r)s''(r)) \le 0. \quad (5)$$

Differentiating (4) with respect to r, we have

$$t''(r) + ps''(r) - 2r\psi^2([s'(r)]^2 + s(r)s''(r)) - 2\psi^2 s(r)s'(r) = 0. \quad (6)$$

Simultaneously considering (5) and (6), we have $s'(r) \le 0$. Furthermore, because the utility function satisfies the Spence-Mirrlees condition (constant sign for $\partial^2 U/\partial s \partial r$), satisfying local incentive-compatible conditions ensures global incentive compatibility. Because the utility function satisfies the Spence-Mirrlees condition, we can observe that the analytical development of the contract mirrors a standard screening problem with the high and low types interchanged in the market. The total rent (consumer surplus) that the vendor will be paying to each consumer is

$$U(r) = t(r) + ps(r) - r\psi^2[s(r)]^2.$$

The local incentive-compatibility constraint can now be written as

$$\frac{dU}{dr} = t'(r) + ps'(r) - 2r\psi^2 s(r)s'(r) - \psi^2[s(r)]^2. \quad (7)$$

As $t'(r) = -ps'(r) + 2\psi^2 rs(r)s'(r)$ from the first-order condition in (4), by employing the envelope theorem, we can find how the rent has to change as consumer type r changes, in order to satisfy incentive constraint given in (7). Hence, the slope of the rent is given as

$$\frac{dU}{dr} = -\psi^2[s(r)]^2.$$

We can now write the vendor's profit function as

$$\pi_{vs-vc} = \int_{\underline{R}}^{\bar{R}} [\alpha\psi s(r) - t(r)] f(r)\, dr, \quad (8)$$

subject to

$$\frac{dU}{dr} = -\psi^2[s(r)]^2, \quad (9)$$

$$s'(r) \le 0, \quad (10)$$

$$U(r) \ge 0. \quad (11)$$

Because $\bar{R}$ is the most privacy-seeking consumer who is served by this contract, from (9) the participation constraint in (11) simplifies to

$$U(\bar{R}) \ge 0. \quad (12)$$

The participation constraint of the most privacy-seeking consumer will be binding, $U(\bar{R}) = 0$. Momentarily ignoring (10) and solving (9), we get $U(\bar{R}) - U(r) = -\psi^2 \int_r^{\bar{R}} [s(x)]^2\, dx$. Because $U(\bar{R}) = 0$, we can simplify this to

$$U(r) = \psi^2 \int_r^{\bar{R}} [s(x)]^2\, dx. \quad (13)$$

Now, using (13) we can rewrite the vendor's profit function in (8) as

$$\pi_{vs-vc} = \int_{\underline{R}}^{\bar{R}} \left[\alpha\psi s(r) + ps(r) - \psi^2 r[s(r)]^2 - \psi^2 \int_r^{\bar{R}} [s(x)]^2\, dx \right] f(r)\, dr. \quad (14)$$

Using Fubini's theorem where the order of integration in iterated integrals can be interchanged, the vendor's profit function in (14) can be rewritten as

$$\pi_{vs-vc} = \int_{\underline{R}}^{\bar{R}} \left[\alpha\psi + p - \psi^2 rs(r) - \psi^2 s(r) \frac{F(r)}{f(r)} \right] s(r) f(r)\, dr. \quad (15)$$

Lemma 3. *In a variable services-coupons contract, both the optimal services* ($s^*_{vs-vc}(r)$) *and coupons* ($t^*_{vs-vc}(r)$) *are decreasing in privacy sensitivity* (r).

Proposition 3. *A high MVI vendor* ($\alpha\psi \ge p[1 + (2/\bar{R}f(\bar{R}))]$) *always finds it optimal to engage in couponing for the entire market whereas a low MVI vendor* ($\alpha\psi \le p$) *does not offer coupons.*

The solution to vendor's optimization of profit function given in (15) gives the optimal services level and coupons as

$$s^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2} \frac{1}{k(r)},$$

$$t^*_{vs-vc}(r) = \frac{\alpha\psi + p}{2\psi^2} \left(\frac{-p}{k(r)} + \frac{(\alpha\psi + p)}{2} \cdot \left(\frac{r}{[k(r)]^2} + \int_r^{\bar{R}} \frac{dx}{[k(x)]^2} \right) \right),$$

where

$$k(r) = \left(r + \frac{1}{h(r)}\right) \quad \forall\, r \in [\underline{R}, \bar{R}].$$

Lemma 3 indicates that as users' privacy sensitivity increases, a vendor offers fewer coupons because of incentive compatibility issues. Thus, counter to intuition, our results show that a vendor uses higher levels of coupons to incentivize convenience seekers to use more personalization services rather than to attract privacy seekers. Proposition 3 shows us that whether a vendor offers coupons depends on the MVI of the vendor relative to the distribution of consumer types in the market. Note that for the entire market to get coupons, it requires a higher MVI

$$\alpha\psi \geq \left(p\left[1 + \frac{2}{\bar{R} f(\bar{R})}\right]\right)$$

for the vendor than in the fixed services fixed coupon case (which only requires $\alpha\psi > p$ because the vendor's value premium has to be sufficiently high to provide coupons to the entire market).

Furthermore, although an intuitive approach might suggest that more privacy-sensitive consumers should be paid larger coupons to incentivize them to consume more services, our analysis shows larger coupons should be given to consumers with low privacy concerns. The economic reasoning behind this result is that if the vendor does not pay larger coupons to the consumers with low privacy concerns, these consumers will misrepresent and declare themselves to be more privacy sensitive than they truly are. In addition, in the fixed services-coupon case, the vendor never serves the entire market and the rent paid declines linearly as privacy concerns increase, whereas in the variable services-coupons case, a large MVI vendor serves the entire market and the rent paid to consumers has a nonlinear decline with increasing privacy concerns. If the vendor does not offer coupons in the fixed services case because his MVI is too small, then in the variable services case also no coupons are offered.

An interesting situation arises when the vendor has a moderate MVI. From Lemma 3, we know that $t^*_{vs-vc}(r)$ is decreasing in r and turns negative for the moderate MVI vendor; negative coupons essentially imply that the vendor has to charge a price from the privacy seekers. Prices cannot be charged in the current personalization services market, i.e., our current setup requires $t(r) \in \mathbb{R}^+$. In this case, the vendor has the option simply to cut off the services offering where $t(r) = 0$. However, this would imply that the last consumer served still enjoys positive utility from personalization services alone, i.e., the most inefficient type served is left with positive surplus—suggesting the possibility of a better solution than that given in Lemma 3. In other words, any optimal solution should consider both instruments (rather than simply continue to offer the same services level while not charging a price) and endogenously determine the last consumer who would be served with the nonprice constraint. Intuitively, we can see that this results in the monotonicity constraint on the services menu to be binding, i.e., $s'^*_{vs-vc}(r) = 0$ for a portion of the market.

Proposition 4. *The constraint* $t^*_{vs-vc}(r) \in \mathbb{R}^+$ (*i.e., no prices are charged in this market*) *requires a vendor with moderate MVI* $(\alpha\psi \in [p, p(1 + (2/\bar{R} f(\bar{R})))])$ *to offer a bunched contract such that*

(a) *consumers defined by* $r \in [\underline{R}, r^*_{vf}]$ *are offered the same contract as given by optimization of profit function given by* (15);

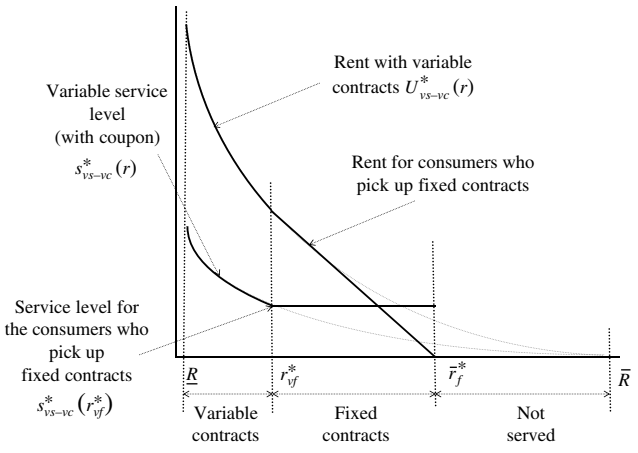
(b) *a fixed services-coupon contract is offered to consumers defined by* $r \in [r^*_{vf}, \bar{r}^*_f]$;

(c) *privacy-sensitive consumers defined by* $r \in [\bar{r}^*_f, \bar{R}]$ *are not served, and the most privacy-sensitive consumer that is served is defined by* $\bar{r}^*_f = (ps^*_{vf}(r^*_{vf}) + t^*_{vf}(r^*_{vf}))/(\psi^2[s^*_{vf}(r^*_{vf})]^2)$, *where*

$$r^*_{vf} = \arg\max_{r_{vf}} \int_{\underline{R}}^{r_{vf}} (\alpha\psi s(r) - t(r)) f(r)\, dr + (\alpha\psi s(r_{vf}) - t(r_{vf})) \int_{r_{vf}}^{\bar{r}^*_f} f(r)\, dr.$$

Proposition 4 suggests that the optimal solution is a combination of fixed and variable contracts (Figure 2) and is a direct consequence of the vendor's inability to charge prices in the market. There are three salient features of this contract: First, the market is not covered, i.e., some high privacy-sensitive consumers are not served even if the marginal cost of serving

Figure 2 Bunched Solution



an additional consumer and the versioning cost is zero. Second, the convenience seekers (with low privacy costs) are unaffected by the nonprice constraint, because they continue to receive the same coupon as might have been available to them if the vendor served the entire market with variable contracts. The middle type consumers, however, receive a fixed contract that extracts all surplus from the consumer type $r = \bar{r}_f^*$, i.e., $U(\bar{r}_f^*) = 0$.

Note that for a nonprice market, the contract given in Proposition 4 is weakly superior to any fixed services or variable services contract. The simple economic intuition behind this observation is that the purely variable and purely fixed contracts are two extreme cases of a market served by a bunched contract. When the vendor has a high MVI, the coupons are always positive for the entire market and Proposition 4 converges to Lemma 3 because of point-wise maximization for profit maximization, where $r_{vf}^* \to \bar{R}$ (the consumer receiving zero surplus is the most privacy-sensitive in the market). Similarly if $r_{vf}^* \to \underline{R}$, and $\bar{r}_f^* \to r_{fs-fc}^*$, then the bunched solution converges to the fixed contract described in Lemma 1. Thus, the bunched contract solution subsumes previous contracts.

The optimality of couponing itself is a function of the vendor's relative MVI. When couponing is not feasible ($\alpha\psi \leq p$), the optimality of fixed services versus variable services contracts depends on the distribution of consumer types in the market, and it is not obvious if the vendor should engage in fixed services-only or variable services-only contracts because there are advantages and disadvantages of both. A fixed services contract has a plus side for the vendor because some consumers are forced to use more services than their optimal level, but on the flip side, to maximize the size of the market served, the vendor offers fewer services than he could have to some consumers (convenience seekers with low r). The variable services-only contract results in a consumer-surplus-maximizing contract to all consumers. For any durable goods market, it is perhaps rare if not impossible that a consumer-surplus-maximizing solution is also optimal from a monopolist's perspective. Interestingly, in this market of NFD goods when the consumers' privacy concerns are highly dispersed, then such a contract may be optimal. The intuition behind this observation is that when consumers are highly dispersed in their privacy concerns, the fixed services-only contract serves a small portion of the market. In such a situation, allowing consumers to choose their own optimal services levels and thereby ensuring that the market is covered, may be a better strategy for the vendor.

Whereas prices are not charged in the personalization services market today, our framework can be extended to analyze the hypothetical situation in which users can be charged for personalization services. Although the convenience seekers get the same contracts independent of the ability of a vendor to charge prices, consumers receiving fixed contracts in Proposition 4 are always worse off than if they would have received variable contracts (as in Lemma 3) because they are forced to pick up the incentive compatible contract of a consumer with $r = r_{vf}^*$, and a whole set of privacy seekers $r \in [\bar{r}_f^*, \bar{R}]$ are not served and therefore get zero surplus. Similarly, the vendor is also worse off because of his inability to charge users: First, he does not serve the entire market and thus gets no information from privacy seekers $r \in [\bar{r}_f^*, \bar{R}]$. Second, he extracts less than the maximum possible surplus from the consumers engaged by fixed contracts because all these consumers are now served with the contract meant for the type $r = r_{vf}^*$.

Intuitively, one might suppose that keeping the personalization services free is beneficial at least to the consumers. However, our analytical framework suggests that this is not true. When the vendor has moderate MVI, consumers and the vendor are better off when the privacy-sensitive consumers pay a price for consuming personalization services, while convenience seekers receive coupons. As the market for preference information is evolving, it is likely that vendors are still fine-tuning their advertising strategies, growing their content networks, and thus have limited ability to exploit consumer information resulting in moderate MVI. In such cases, pricing personalization services for privacy seekers might serve to increase consumer surplus as well as social welfare.

4. Discussion

The focus of our work is to design mechanisms for the delivery of personalization services in a market for information. We develop both fixed and variable services contracts for a heterogeneous market, with and without the option of offering coupons. We present four primary results. Proposition 1 provides a preliminary characterization of a vendor who cannot vary personalization (for example, who provides a personalization toolbar of fixed functionality) and might consider offering a coupon as an incentive to adopt the toolbar. The result provides an intuitive separation condition between when a coupon is offered and when it is not; this depends on whether the MVI is larger or smaller than the ratio of the marginal "direct value of personalization" and the "difficulty of personalization." Proposition 2, is a significant result because it establishes that when allowing variable service levels in the absence of variable couponing, all

consumers (rather than just the highest type) consume at a consumer-surplus-maximizing level. This departure from the standard screening contract is a consequence of both key differences characterizing the market setting: the seller not having a sufficiently flexible "payment" instrument that is distinct from the consumers' payment of personal information, and the NFD property. It is important to note that this may not be the efficient solution; for a sufficiently high MVI, total surplus could be maximized by a consumer by consuming more than surplus-maximizing service level. However, in the absence of variable couponing, the latter cannot be incentive compatible. There are a number of reasons why sellers may not be able to offer variable coupons (transaction costs, lack of consumer familiarity, lack of a good micropayment system), and therefore, the result is perhaps more managerially relevant. Finally, Proposition 3, characterizes the vendor's optimal contract in the most general case of variable couponing and variable services and provides three regions—low, medium, and high MVI—that have contracts of different kinds. Interestingly, vendors with a low MVI simply use the contract specified in Proposition 2, and it takes a sufficiently high level of MVI for the fully revealing contract to be revenue maximizing. A key finding is that the direct compensation for information is inversely related to privacy sensitivity owing to incentive-compatibility issues.

Our results suggest that in the absence of couponing, a vendor's choice between fixed and variable services-only contracts depends on the degree of heterogeneity of consumers. Consistent with intuition, a fixed services-only contract performs badly with high variance in privacy costs because a one-size-fits-all approach is inappropriate when consumers are highly differentiated. Interestingly, a variable services-only contract is superior in such cases even if these contracts are consumer-welfare maximizing and the vendor a monopolist. The advantage of such variable contracts is more pronounced in a market that is dominated by convenience seekers, because these consumers prefer a high number of services and the relative benefit to the vendor from forcing consumers to use more than their optimal services level (as a fixed contract would do) is small.

The NFD property of personalization services has certain interesting implications on the nature of rent paid to the consumers. For any consumer served by the variable services-coupons contract, the services consumed are greater than his surplus-maximizing-services level. Hence, for a given consumer type, the surplus (from personalization services alone) is higher from a contract meant for a more privacy-sensitive neighbor. This surplus is first increasing in the "distance" between the two types; however, after a certain point, when the contract meant for more privacy-sensitive types equals the surplus maximizing contract for the given consumer type, this surplus begins to decrease. Therefore, the rent to be paid (purely as a function of surplus from services) to prevent a given consumer type from picking up the lower services contract meant for more privacy-sensitive user, is first increasing and then decreasing. This causes the coupons to be first positive and then negative (from convenience seekers to privacy seekers) so as to ensure that the overall rents are decreasing in types. However, the constraint of nonnegative payments (inability to charge prices) requires certain vendor types to forgo a fully nonlinear menu and opt for a bunched contract.

As long as the vendor is considering couponing, the bunched solution is weakly superior because it encompasses other contracts in its construction. Under differing market conditions, only the segment of consumers receiving variable and fixed contracts (or no contracts at all) will vary. In traditional vertically segmented markets, a shutdown condition comes into play (where only contracts for the high type are offered) because of large differences in types (Mussa and Rosen 1978). Analogously, when convenience seekers dominate a market and there are not enough privacy seekers to warrant rent to the convenience seekers, the fixed portion of the bunched solution is increasingly important. Of course the vendor profits are the highest when a market is concentrated and dominated by convenience seekers. Note that serving only a part of the market is an optimal vendor strategy under certain conditions, even though service provision is costless.

4.1. Implications for Theory

From a theoretical point of view, this work contributes to research in mechanism design for information goods. Sundararajan (2004) observes that the zero marginal cost of producing digital goods, along with some marginal cost in administering them, results in a bunched optimal contract. Sundararajan (2004) also points out (p. 1670) that if the marginal cost component does not exist, then a fully revealing, purely usage-based contract becomes optimal, along the lines of well-known results of Maskin and Riley (1984). Our results show that even when there is no marginal cost to the vendor, a bunched contract can be optimal owing to the NFD nature of the services and nonprice market characteristics. The economic intuition behind this result is that a high-type consumer's surplus from misrepresenting himself is nonmonotonically increasing in services resulting in negative coupons, and thus, the nonprice nature of the market requires the vendor to consider fixed contracts. Further, in Sundararajan (2004), a fixed but buffet contract (the full

menu) is offered to the high types because there is free disposal. However, our results suggest that under NFD property, fixed contracts are suitable to extract surplus from more privacy-seeking types.

In the introduction to our work we observed some similarities between a monopolist's screening problem with hidden ability and ours while noting the two distinct differences: NFD property of personalization services, and restriction on pricing. As our results show, the combined effect of these two differences can lead to schedules that are not strictly nonlinear because coupons change sign over the consumer types. Interestingly, in spite of the unusual shape of the utility function (nonmonotonic concave), a self-selecting menu can still be developed because the preferences satisfy the Spence-Mirrlees condition.

There are structural similarities between our model and that of regulator-firm models where firms create pollution or environmental damage to the society. For example, in a recent work on incentives for pollution-creating firms (Levi and Nault 2004), the firms' profit function (analogous to our consumers' utility function) is nonmonotonic concave in output. This model imposes a constraint that subsidies (taxes) cannot be negative (positive) and concludes (under certain conditions) that a simple lump-sum subsidy is superior to an output-based one. Our analyses suggest that a globally optimal superior solution is one in which the regulator offers a bunched contract where a portion of the market picks up the lump-sum subsidy, and vendors with better technology pick up the production-based subsidy. The intuition supporting this solution is that the regulator's constraint on nonpositive taxes or nonnegative subsidies is similar to our market constraint that no prices can be charged for personalization services.

4.2. Implications for Practice

Several managerial implications result from our analyses. Our findings show that a fixed-length toolbar that forces consumers to use a defined set of services is a superior approach only under narrowly defined circumstances. If convenience seekers dominate a market, Web portals are better off by offering a toolbar of maximum length and allowing consumers to turn off features based on their privacy concerns. Currently, most portals including Yahoo! and Google pursue this strategy. With regards to coupons, our results clearly indicate that coupons should be employed to further incentivize consumers who are comfortable with personalization and sharing information, rather than to incentivize privacy seekers. Thus, couponing is more effective as a strategy to extract surplus than as one to acquire new users.

Although a majority of portals are going in the direction of variable services levels for their toolbars, Amazon.com is pursuing a different approach today. In the pretoolbar era, Amazon introduced discount coupons based upon consumers' clicking through its Gold Box services and sharing information. This feature has since been dropped, because this was an ill-conceived combination of fixed coupons with variable services, i.e., the consumer was allowed to choose his level of personalization but everyone received the same benefits. Currently, through A9.com's toolbar and services, Amazon.com uses a fixed services contract where its privacy policy clearly states that once the toolbar is downloaded a fixed amount of information will be monitored and subsequent usage will be personalized. More recently, Amazon.com introduced coupons for using this service, e.g., an instant reward of 1.57% (which Amazon calls $\pi/2\%$) of the purchase amount after adequate usage of A9.com services. Along the lines of our discussion of a vendor sharing surplus with consumers through coupons, Amazon.com (2004) clearly specifies, "How can we afford this?—Sponsored links revenue—from the small text-based ads on A9.com and Amazon.com search results pages—will help offset costs we incur through the Instant Reward promotion. With our automatic $\pi/2\%$ Instant Reward, we are effectively sharing with you some of the money we collect from sponsored links, i.e., sharing the pi." Although, at this time Amazon.com will not reveal the level of services usage for which this coupon will become effective, one can observe that this is the initial stage of constructing a combination of fixed and variable contracts.

Two other factors can affect how the market might evolve over time. First, the technology parameter may improve as more personalization services can be provided for a given amount of information. Second, consumers' distribution of privacy cost may also evolve. However, the direction of this change is a function of how service providers treat consumers over time. If the vendors can engender trust in how consumer information is used, we may see skewness toward convenience seekers. On the other hand, if there is repeated evidence of misuse of information, more consumers may turn into privacy seekers, making it increasingly difficult for the vendor to acquire information. In this regard, our work suggests that charging for personalization services for privacy seekers and providing coupons for convenience seekers may be a socially beneficial strategy.

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Appendix A. Summary of Notations and Results

Symbol	Definition
p	Consumer's marginal value for personalization services
α	Vendor's marginal value of information
ψ	Technology that determines how many personalized services can be created for a unit of information
s	Personalization services
t	Coupons
$r \in [\underline{R}, \bar{R}]$	Consumer privacy cost coefficient—distributed with pdf $f(\cdot)$ and cdf $F(\cdot)$
$u(r, s)$	Consumers' utility from personalization services
$U(r)$	Consumers' utility from personalization services and coupons
$s_c^*(r)$	Surplus maximizing services
$s_c^0(r)$	Break-even services
$\{s, t\}$	Fixed contracts
$\{s(r), t(r)\}$	Variable contracts
Subscripts	
$fs\text{–}nc$	Fixed services-only
$fs\text{–}fc$	Fixed services-coupon
$vs\text{–}nc$	Variable services-only
$vs\text{–}vc$	Variable services-coupons

Appendix B. Proofs of Lemmas and Propositions

Proof of Lemma 1 ($fs\text{–}nc$ and $fs\text{–}fc$). Consumers that derive positive utility from the offered fixed services level will contract and let r_{fs-nc} be the consumer who is indifferent between taking the contract and not participating. Because $\partial u(r, s)/\partial r < 0|_{\forall s>0}$, we can conclude that all consumers whose privacy sensitivity is less than $r = p/\psi^2 s_{fs-nc}$ will participate, i.e., the market given by $r \in [\underline{R}, r_{fs-nc}]$ will be served. Taking the first-order condition (FOC) of the objective function, we have

$$\frac{\partial}{\partial s}\alpha\psi s\int_{\underline{R}}^{r_{fs-nc}} f(r)\,dr = \frac{\partial}{\partial s}\alpha\psi s F\left(\frac{p}{\psi^2 s}\right) \Rightarrow \alpha\psi F\left(\frac{p}{\psi^2 s}\right) + \alpha\psi s\left(\frac{-p}{\psi^2 s^2}\right) f\left(\frac{p}{\psi^2 s}\right) = 0$$

$$= \psi F\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi s} f\left(\frac{p}{\psi^2 s}\right) = 0.$$

Thus the optimal services level s_{fs-nc}^* is

$$s_{fs-nc}^* = \frac{p}{\psi^2}\frac{f(p/\psi^2 s_{fs-nc}^*)}{F(p/\psi^2 s_{fs-nc}^*)} = \frac{p}{\psi^2} h(r_{fs-nc}^*).$$

Checking for the second-order condition (SOC)

$$\pi''_{fs-nc} = \frac{\partial^2}{\partial s^2}\alpha\psi s\int_{\underline{R}}^{r_{fs-nc}} f(r)\,dr$$

$$= \frac{\partial}{\partial s}\alpha\psi\left[F\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi^2 s} f\left(\frac{p}{\psi^2 s}\right)\right]$$

$$= \alpha\psi\left[-\frac{p}{\psi^2 s^2} f\left(\frac{p}{\psi^2 s}\right) - \frac{p}{\psi^2 s}\left(-\frac{p}{\psi^2 s^2}\right) f'\left(\frac{p}{\psi^2 s}\right) + \frac{p}{\psi^2 s^2} f\left(\frac{p}{\psi^2 s}\right)\right] = \frac{\alpha p^2}{\psi^3 s^3} f'(r_{fs-nc}).$$

For $\pi''_{fs-nc}(s_{fs-nc}^*) < 0$, we need to have $(\alpha p^2/\psi^3 s^3) f'(r_{fs-nc}^*) < 0$. Note that f is a unimodal distribution in interval $[\underline{R}, \bar{R}]$, hence, $\pi''_{fs-nc}(s_{fs-nc}^*) < 0$ if $r_{fs-nc}^* >$ mode of f. Hence, the optimal profit is

$$\pi_{fs-nc}^* = \alpha\psi\int_{\underline{R}}^{r_{fs-nc}^*} s_{fs-nc}^* f(r)\,dr = \frac{\alpha p}{\psi}\frac{f(p/\psi^2 s_{fs-nc}^*)}{F(p/\psi^2 s_{fs-nc}^*)} F\left(\frac{p}{\psi^2 s_{fs-nc}^*}\right) = \frac{\alpha p}{\psi} f(r_{fs-nc}^*).$$

When the vendor offers fixed coupon t along with fixed services, all consumers whose privacy sensitivity is less than $r_{fs-fc} = (ps + t)/\psi^2 s^2$ will participate, hence, the portion of the market given by $r \in [\underline{R}, r_{fs-fc}]$ will be served. For the optimal solutions we need to consider FOCs of the profit function with respect to (w.r.t.) s and t and simultaneously solve for optimal values.

The FOC w.r.t. s gives

$$\frac{\partial}{\partial s}\int_{\underline{R}}^{(ps+t)/\psi^2 s^2} [\alpha\psi s - t] f(r)\,dr = \frac{\partial}{\partial s}[\alpha\psi s - t] F\left(\frac{ps+t}{\psi^2 s^2}\right) = 0.$$

Simplifying, we have

$$\alpha\psi^3 F\left(\frac{ps+t}{\psi^2 s^2}\right) - [\alpha\psi s - t]\left[\frac{p}{s^2} + \frac{2t}{s^3}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right) = 0. \quad \text{(B1)}$$

The FOC w.r.t. t gives

$$\frac{\partial}{\partial t}\int_{\underline{R}}^{(ps+t)/\psi^2 s^2} [\alpha\psi s - t] f(r)\,dr = \frac{\partial}{\partial t}[\alpha\psi s - t] F\left(\frac{ps+t}{\psi^2 s^2}\right) = 0.$$

Simplifying, we have

$$-F\left(\frac{ps+t}{\psi^2 s^2}\right) + \left[\frac{\alpha\psi s - t}{\psi^2 s^2}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right) = 0. \quad \text{(B2)}$$

Now solving Equations (B1) and (B2) simultaneously, we have

$$\frac{\alpha\psi}{s^2} - \left[\frac{p}{s^2} + \frac{2t}{s^3}\right] = [\alpha\psi - p]s - 2t = 0 \Rightarrow t = \frac{[\alpha\psi - p]s}{2}.$$

Hence, we have

$$s_{fs-fc}^* = \frac{p + \alpha\psi}{2\psi^2} h(r_{fs-fc}^*), \quad t_{fs-fc}^* = \frac{[\alpha\psi - p]s_{fs-fc}^*}{2}. \quad \text{(B3)}$$

Checking for the SOC w.r.t. s, we have

$$\frac{\partial^2 \pi_{fs-fc}}{\partial s^2} = \frac{\partial}{\partial s}\left[\alpha\psi F\left(\frac{ps+t}{\psi^2 s^2}\right) - \frac{1}{\psi^2}[\alpha\psi s - t]\left[\frac{p}{s^2} + \frac{2t}{s^3}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right)\right]$$

$$= -\frac{2\alpha}{\psi}\left[\frac{p}{s^2} + \frac{2t}{s^3}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right) - \left[\frac{\alpha\psi s - t}{\psi^2}\right] \cdot \left[\left[-\frac{2p}{s^3} - \frac{6t}{s^4}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right) - \left[\frac{p}{s^2} + \frac{2t}{s^3}\right]^2 f'\left(\frac{ps+t}{\psi^2 s^2}\right)\right].$$

We also know that the optimal coupon that should accompany the offered services level is given by $t_{fs-fc}^* = [\alpha\psi - p]s_{fs-fc}^*/2$. Also note that

$$\frac{ps + t_{fs-fc}^*}{[\psi s_{fs-fc}^*]^2} = r_{fs-fc}^*,$$

where r^*_{fs-fc} represents the last customer served. Simplifying the SOC, we have

$$\left.\frac{\partial^2\pi}{\partial s^2}\right|_{s^*_{fs-fc},\,t^*_{fs-fc}} = -\frac{f(r^*_{fs-fc})}{2[\psi s^*_{fs-fc}]^2}[\alpha\psi - p]^2 + \frac{\alpha^2[\alpha\psi+p]}{2[s^*_{fs-fc}]^3} f'(r^*_{fs-fc}).$$

For the interior solution, the first term is always negative, and the second term is negative if $r^*_{fs-fc} >$ mode of f, because f is a unimodal distribution in the interval $[\underline{R}, \bar{R}]$.

Similarly, checking for the SOC w.r.t. t, we have

$$\frac{\partial^2\pi_{fs-fc}}{\partial t^2} = \frac{\partial}{\partial t}\left[-F\left(\frac{ps+t}{\psi^2 s^2}\right) + \left[\frac{\alpha\psi s - t}{\psi^2 s^2}\right] f\left(\frac{ps+t}{\psi^2 s^2}\right)\right]$$
$$= \frac{1}{\psi^2 s^2}\left[-2f\left(\frac{ps+t}{\psi^2 s^2}\right) + \left[\frac{\alpha\psi s - t}{\psi^2 s^2}\right] f'\left(\frac{ps+t}{\psi^2 s^2}\right)\right].$$

We know that at the optimal $t^*_{fs-fc} = [\alpha\psi - p]s^*_{fs-fc}/2$, hence

$$\left.\frac{\partial^2\pi_{fs-fc}}{\partial t^2}\right|_{s^*_{fs-fc},\,t^*_{fs-fc}} = -\frac{2}{[\psi s^*_{fs-fc}]^2} f(r^*_{fs-fc}) + \left[\frac{p+\alpha\psi}{2\psi[\psi s^*_{fs-fc}]^3}\right] f'(r^*_{fs-fc}).$$

We see that the second term is negative for any solution $r^*_{fs-fc} >$ mode of f, because f is a unimodal distribution in the interval $[\underline{R}, \bar{R}]$. Furthermore, by substituting the optimal values in the profit function, we get

$$\pi^*_{fs-fc} = \left[\frac{\alpha\psi+p}{2\psi}\right]^2 f(r^*_{fs-fc}).$$

From (B3), optimal coupon value is linearly increasing in optimal fixed services offered by the vendor. □

Proof of Proposition 1. The first part of this proposition is rather easy to see because $t^*_{fs-fc} > 0$ only when $\alpha\psi > p$ and indeed the services levels of Lemma 1, with and without coupons, converge when $\alpha\psi = p$. Furthermore, $r^*_{fs-nc} = p/\psi^2 s^*_{fs-nc}$ and $r^*_{fs-fc} = (\alpha\psi+p)/2\psi^2 s^*_{fs-fc}$. Substituting for s^*_{fs-nc} and s^*_{fs-fc}, respectively, we get $r^*_{fs-nc} = 1/(h(r^*_{fs-nc}))$ and $r^*_{fs-fc} = 1/(h(r^*_{fs-fc}))$. Because the function h is common to both cases, the 45 degree line can cut $1/h$ at only one point, and hence, $r^*_{fs-nc} = r^*_{fs-fc}$. We can also see that the vendor's profits are clearly higher with couponing. We also know that $u(r^*_{fs-nc}) = U(r^*_{fs-fc}) = 0$. Let $r^*_{fs} = r^*_{fs-nc} = r^*_{fs-fc}$. The surplus for the most convenience-seeking consumer ($\underline{R}$) in *fs–nc* case can be written as

$$u_{fs-nc}(\underline{R}) = \frac{p^2}{\psi^2} h(r^*_{fs}) - \underline{R}\frac{p^2}{\psi^2} h^2(r^*_{fs}) = \frac{p^2}{\psi^2} h(r^*_{fs})[1 - h(r^*_{fs})].$$

Similarly, the surplus for the same consumer in the *fs–fc* case can be written as

$$U_{fs-fc}(\underline{R}) = \frac{[\alpha\psi - p]}{2} s^*_{fs-fc} + p s^*_{fs-fc} - \underline{R}\psi^2[s^*_{fs-fc}]^2$$
$$= \frac{[\alpha\psi+p]}{2} s^*_{fs-fc} - \underline{R}\psi^2[s^*_{fs-fc}]^2$$
$$= \frac{[\alpha\psi+p]}{2}\frac{[\alpha\psi+p]}{2\psi^2} h(r^*_{fs}) - \underline{R}\psi^2\frac{[\alpha\psi+p]^2}{4\psi^4} h^2(r^*_{fs})$$
$$= \frac{[\alpha\psi+p]^2}{4\psi^2} h(r^*_{fs})[1 - \underline{R}h(r^*_{fs})].$$

When couponing is an optimal strategy ($\alpha\psi > p$) in effect, then we can clearly see that $U_{fs-fc}(\underline{R}) > u_{fs-nc}(\underline{R})$. We also know that both U_{fs-fc} and u_{fs-nc} are linear in r; hence $U_{fs-fc}(r) > u_{fs-nc}(r)$ for all $r \in [\underline{R}, r^*_{fs})$, i.e., every consumer but the last (r^*_{fs}) will get a greater surplus with couponing and the last served consumer gets zero surplus in both cases. Because the profits and consumer surplus are higher with couponing, the social welfare is also higher in this case.

Furthermore, if some fixed services s_f is offered with some variable coupons $t(r)$, the consumer will always misrepresent his privacy cost coefficient such that his utility $ps_f - r\psi^2 s_f^2 + t(r)$ is maximized. In fact, all of them will declare an $\tilde{r} = t'^{-1}(0)$, i.e., at which $t(r)$ is maximized. □

Proof of Lemma 2 and Proposition 2 (*vs–nc*). In the absence of any coupons, the only instrument that the vendor has in the variable case is the services level. Ideally, the vendor would like to offer the break-even services $s^0_c(r) = p/\psi^2 r$, where he extracts full surplus from the consumers. However, if he declares this to be his services offering, all consumers of type r will misrepresent their type to be some $\tilde{r} = 2r$, such that they get their surplus maximizing services level of $p/2\psi^2 r$. In general, for any $s(r)$ offered by the vendor, the consumers will declare their type such that $s(\tilde{r}) = p/2\psi^2 r$, or $\tilde{r} = s^{-1}(p/2\psi^2 r)$. Hence, the vendor has no option but to offer $s(r) = p/2\psi^2 r$. If the vendor were to consider a variable services, fixed coupon, the consumer will misrepresent his true privacy concerns and declare the same $\tilde{r}$ as above and will get the coupons as well. □

Proof of Lemma 3 (*vs–vc*). The vendor's profit function is given as

$$\pi_{vs-vc} = \int_{\underline{R}}^{\bar{R}} \left[\alpha\psi + p - \psi^2 r s(r) - \psi^2 s(r)\frac{F(r)}{f(r)}\right] s(r) f(r)\, dr.$$

Employing point-wise maximization, we need to only maximize the portion inside the integral w.r.t. $s(r)$. This gives

$$s^*_{vs-vc}(r) = \frac{\alpha\psi+p}{2\psi^2[r + (F(r)/f(r))]} = \frac{\alpha\psi+p}{2\psi^2}\frac{1}{[r + (1/h(r))]}. \quad \text{(B4)}$$

Now we check for the condition that was ignored earlier (Equation (10) in this paper). Differentiating $s^*_{vs-vc}(r)$,

$$\frac{\partial}{\partial r} s^*_{vs-vc}(r) = -\frac{\alpha\psi+p}{2\psi^2}\frac{[1 - (h'(r)/h^2(r))]}{[r + (1/h(r))]^2}.$$

Because $h'(r) \le 0$ (from standard monotone hazard rate assumption explicated earlier in this paper) and because $h(r) > 0$, $(\partial/\partial r)s^*_{vs-vc}(r)$ is always negative thus satisfying the condition in (10) (in this paper) we had ignored earlier.

From (13) in this paper, we know

$$U(r) = \psi^2 \int_r^{\bar{R}} \frac{[\alpha\psi+p]^2 h^2(x)}{4\psi^4[1 + x h(x)]^2}\, dx.$$

Thus, we have

$$U^*_{vs-vc}(r) = \left[\frac{\alpha\psi+p}{2\psi}\right]^2 \int_r^{\bar{R}} \frac{dx}{[(1/h(x)) + x]^2}.$$

Applying the optimal values and simplifying the rent equation below,

$$U(r) = t(r) + p s(r) - r\psi^2[s(r)]^2,$$

we get

$$t(r) = -p\frac{\alpha\psi+p}{2\psi^2}\frac{h(r)}{[1+rh(r)]} + r\frac{[\alpha\psi+p]^2}{4\psi^2}\frac{h^2(r)}{[1+rh(r)]^2} + \left[\frac{\alpha\psi+p}{2\psi}\right]^2\int_r^{\bar{R}}\frac{h^2(x)}{[1+xh(x)]^2}\,dx,$$

which gives the optimal coupon in the variable case as

$$t^*_{vs-vc}(r) = \frac{\alpha\psi+p}{2\psi^2}\left[\frac{-p}{[r+(1/h(r))]} + \frac{[\alpha\psi+p]}{2}\cdot\left[\frac{r}{[r+(1/h(r))]^2} + \int_r^{\bar{R}}\frac{dx}{[r+(1/h(r))]^2}\right]\right]. \quad \text{(B5)}$$

From (B4) and (B5) and by the fact that $dU/dr = -\psi^2[s(r)]^2$, we have Lemma 3. □

Proof of Proposition 3. We know that for the most privacy-sensitive consumer $(\bar{R})U(\bar{R}) = 0$. Because $U(\bar{R}) = u(\bar{R}) + t(\bar{R})$, if the consumer enjoys positive utility from usage of personalization services alone, i.e., if $u(\bar{R}) > 0$, we know that $t(\bar{R})$ must be negative. Checking for the utility of consumer of type $\bar{R}$, we have $u(\bar{R}) = ps(\bar{R}) - r\psi^2 s^2(\bar{R}) = s(\bar{R})[p - r\psi^2 s(\bar{R})]$. So if $t(\bar{R})$ has to be negative, then it will suffice to show that $p - r\psi^2 s(\bar{R}) > 0$, i.e.,

$$p - r\psi^2 s(\bar{R}) > 0 \;\Rightarrow\; p > \bar{R}\psi^2\frac{\alpha\psi+p}{2\psi^2}\left[\bar{R} + \frac{1}{h(\bar{R})}\right]^{-1}.$$

This implies

$$p\left[1 + \frac{2}{\bar{R}h(\bar{R})}\right] > \alpha\psi. \quad \text{(B6)}$$

Equation (B6) implies that if $\alpha\psi > p[1 + (2/\bar{R}h(\bar{R}))]$, then $t^*_s(\bar{R}) > 0$.

Applying a similar approach to ensuring $U(\underline{R}) > 0$, i.e., at least the least privacy-sensitive consumer gets positive surplus, we need to ensure that

$$\alpha\psi > p\left[1 + \frac{2}{\underline{R}h(\underline{R})}\right]. \quad \text{(B7)}$$

Because $1/h(\underline{R}) = F(\underline{R})/f(\underline{R})$ and $F(\underline{R}) = 0$, we get $U(\underline{R}) > 0$ if and only if $\alpha\psi > p$. Combining Equations (B6) and (B7), and because $h(\bar{R}) = f(\bar{R})$ as $F(\bar{R}) = 1$, we get conditions described in Proposition 3. □

Proof of Proposition 4. To derive the optimal bunched solution, let the vendor offer a variable services contract $\{s_{vf}(r), t_{vf}(r)\}$ to consumers defined by $r \in [\underline{R}, \ldots, r_{vf}]$, i.e., $(s'_{vf}(r) < 0, t'_{vf}(r) < 0)$ and a fixed contract to consumers defined by $r \in [r_{vf}, \ldots, \bar{r}_f]$, i.e., $((s'_{vf}(r) = 0, t'_{vf}(r) = 0))$:

$$r^*_{vf} = \arg\max_{r_{vf}} \int_{\underline{R}}^{r_{vf}} [\alpha\psi s(r) - t(r)]f(r)\,dr + [\alpha\psi s(r_{vf}) - t(r_{vf})]\int_{r_{vf}}^{\bar{r}^*_f} f(r)\,dr. \quad \text{(B8)}$$

We employ point-wise maximization for the first integral in the equation above; we will arrive at the same menu of contracts as in Lemma 3, i.e., $\{s^*_{vf}(r), t^*_{vf}(r)\}$ will be the same as $\{s^*_{vs-vc}(r), t^*_{vs-vc}(r)\}$ albeit applied to different limits. Thus, substituting the optimal menu in (B8), we can write

$$r^*_{vf} = \arg\max_{r_{vf}} \left[\frac{\alpha\psi+p}{2\psi}\right]^2\int_{\underline{R}}^{r_{vf}}\frac{dr}{[rf(r)+F(r)]} + [\alpha\psi s(r_{vf}) - t(r_{vf})]\int_{r_{vf}}^{\bar{r}^*_f} f(r)\,dr, \quad \text{(B9)}$$

subject to

$$U(\bar{r}^*_f) = ps^*_{vf}(r_{vf}) - \bar{r}^*_f\psi^2 s^{*2}_{vf}(r_{vf}) + t^*_{vf}(r_{vf}) = 0. \quad \text{(B10)}$$

Equation (B10) gives $\bar{r}^*_f = (ps^*_{vf}(r_{vf}) + t^*_{vf}(r_{vf}))/\psi^2 s^{*2}_{vf}(r_{vf})$. Hence, we can write (B9) and maximized profits as

$$r^*_{vf} = \arg\max_{r_{vf}} \left[\frac{\alpha\psi+p}{2\psi}\right]^2\int_{\underline{R}}^{r_{vf}}\frac{dr}{[rf(r)+F(r)]} + [\alpha\psi s^*_{vf}(r_{vf}) - t^*_{vf}(r_{vf})]\cdot\left[F\left(\frac{ps^*_{vf}(r_{vf}) + t^*_{vf}(r_{vf})}{\psi^2 s^{*2}_{vf}(r_{vf})}\right) - F(r_{vf})\right],$$

$$\pi^*_{vf} = \left[\frac{\alpha\psi+p}{2\psi}\right]^2\int_{\underline{R}}^{r^*_{vf}}\frac{dr}{[rf(r)+F(r)]} + [\alpha\psi s^*_{vf}(r^*_{vf}) - t^*_{vf}(r^*_{vf})]\cdot\left[F\left(\frac{ps^*_{vf}(r^*_{vf}) + t^*_{vf}(r^*_{vf})}{\psi^2 s^{*2}_{vf}(r^*_{vf})}\right) - F(r^*_{vf})\right]. \quad \square$$

References

Amazon.com. 2004. What is the A9 Instant Reward? Retrieved July 21, 2006, http://www.amazon.com/gp/xs/sharethepi.html.

Baron, D. P., R. B. Myerson. 1982. Regulating a monopolist with unknown costs. *Econometrica* **50**(4) 911–930.

Bishop, T. 2008. Microsoft shifts strategy with "cashback": Shoppers who use Live Search will get a bonus. *Seattle Post-Intelligencer* (May 26), http://articles.sfgate.com/2008-05-26/business/17153713_1_microsoft-search-engine-search-leader-google-new-search-service.

Chellappa, R. K., S. Shivendu. 2006. A model of advertiser—Portal contracts: Personalization strategies under privacy concerns. *Inform. Tech. Management* **7**(1) 7–19.

Chellappa, R. K., S. Shivendu. 2007. An economic model of privacy: A property rights approach to regulatory choices for online personalization. *J. Management Inform. Systems* **24**(3) 193–226.

Cheng, H. K., G. J. Koehler. 2003. Optimal pricing policies of Web-enabled application services. *Decision Support Systems* **35**(3) 259–272.

Culnan, M. J., P. K. Armstrong. 1999. Information privacy concerns, procedural fairness, and impersonal trust: An empirical investigation. *Organ. Sci.* **10**(1) 104–115.

Geng, X., M. B. Stinchcombe, A. B. Whinston. 2005. Bundling information goods of decreasing value. *Management Sci.* **51**(4) 662–667.

Jain, S., P. K. Kannan. 2002. Pricing of information products on online servers: Issues, models, and analysis. *Management Sci.* **48**(9) 1123–1142.

Levi, M. D., B. R. Nault. 2004. Converting technology to mitigate environmental damage. *Management Sci.* **50**(8) 1015–1030.

Maskin, E., J. Riley. 1984. Monopoly with incomplete information. *RAND J. Econom.* **15**(2) 171–196.

Mendelson, H., S. Whang. 1990. Optimal incentive-compatible priority pricing for the M/M/1 queue. *Oper. Res.* **38**(5) 870–883.

Murthi, B. P. S., S. Sarkar. 2003. The role of the management sciences in research on personalization. *Management Sci.* **49**(10) 1344–1362.

Mussa, M., S. Rosen. 1978. Monopoly and product quality. *J. Econom. Theory* **18**(2) 301–317.

Nahata, B., S. G. Kokovin, E. Zhelobodko. 2006. Package sizes, tariffs, quantity discount and premium. Working paper, http://ssrn.com/abstract=909295.

Raghu, T. S., P. K. Kannan, H. R. Rao, A. B. Whinston. 2001. Dynamic profiling of consumers for customized offerings over the Internet: A model and analysis. *Decision Support Systems* **32**(2) 117–134.

Repullo, R. 1987. The existence of equilibrium without free disposal in economies with transaction costs and incomplete markets. *Internat. Econom. Rev.* **28**(2) 275–290.

Resnick, P., H. R. Varian. 1997. Recommender systems. *Comm. ACM* **40**(3) 56–58.

Rothwell, G., J. Rust. 1997. On the optimal lifetime of nuclear power plants. *J. Bus. Econom. Statist.* **15**(2) 195–208.

Rust, R. T., P. K. Kannan, N. Peng. 2002. The customer economics of Internet privacy. *J. Acad. Marketing Sci.* **30**(4) 455–464.

Sandholm, T., S. Suri. 2003. BOB: Improved winner determination in combinatorial auctions and generalizations. *Artificial Intelligence* **145**(1–2) 33–58.

Sandholm, T., S. Suri, A. Gilpin, D. Levine. 2002. Winner determination in combinatorial auction generalizations. *AAMAS'02: Proc. First Internat. Joint Conf. on Autonomous*, ACM, New York, 69–76.

Shapiro, C., H. R. Varian. 1998. Information rules. Working paper, Harvard Business School Press, Boston.

Sundararajan, A. 2004. Nonlinear pricing of information goods. *Management Sci.* **50**(12) 1660–1673.

Volokh, E. 2000. Personalizaton and privacy. *Comm. ACM* **43**(8) 84–88.